On the Matter of Aggregate Models for Syllogistic Reasoning: A Transitive Set-Based Account for Predicting the Population

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Syllogistic Reasoning

All scientists are gods
Some gods are immortal
What, if anything, follows?

- Reasoning is a core skill of human cognition
- Core domain: syllogisms, i.e., categorical quantified assertions
Example: Atmosphere Heuristic

All scientists are gods
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Logic: No Valid Conclusion

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- Theories try to capture the deviations from logic
- Example: Atmosphere heuristic\(^1\) predicts quantifier
  - by merging quantity and polarity
  - ... but no statement about the direction

### Theories of Syllogistic Reasoning (Khemlani & Johnson-Laird, 2012)

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<th>Formal Rules</th>
<th>Diagrams, Sets &amp; Models</th>
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- Meta-analysis demonstrates: no single best performing theory
- Heuristic approaches perform worse than model-based approaches
Covering the Most Frequently Given Answer

Dataset
- Khemlani2012
- Ragni2016

MFA Coverage

Matching | PHM | Atmosphere | PSYCOP | Conversion | Verbal Models | Mental Models
--- | --- | --- | --- | --- | --- | ---

0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0
Are simple heuristic strategies simply *insufficient* for predicting human syllogistic reasoning?
Research Question

Are simple heuristic strategies simply *insufficient* for predicting human syllogistic reasoning?

Can we identify *simple mechanisms* that explain inferences?
Heuristic Principles

- We need to identify fundamental principles of heuristics
- Requirements for good heuristics, they
  - Should work in many practical situations
    (logically valid when applied correctly)
  - Should not require deep reasoning process (akin to pattern matching)
  - Should leave room for illogical inferences
    (application in unwarranted cases)
Transitivity is a core principle and good heuristic:

1. Works in practice:
   - Basic principle for making inferences

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3. Room for illogical inferences:
   - Transitivity is often applied in unjustified cases (pseudo-transitivity)\(^2\)
   - Participants might force a task into a transitive shape

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Syllogistic Domain

- Total of 64 problems consisting of
  - 4 quantifiers (All, Some, Some ... not, None)
  - 4 figures depending on arrangement of terms (A, B, C)

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<td>B-C</td>
<td>C-B</td>
<td>C-B</td>
<td>B-C</td>
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- Nine possible conclusions:

Eight conclusions relating end terms (A, C) and “No Valid Conclusion” (NVC)
The TransSet Model

1. **Determine direction**
   Search for a transitive path and determine the direction of the conclusion

2. **Determine quantifier**
   Propagate a set along the path
Figure 1:

All A are B, Some B are C

Transitive path directly available (A-B-C)

Analogously possible for Figure 2 (C-B-A)

Directly yields A-C (Figure 1) and C-A direction (Figure 2)
Determine Direction: Finding a Transitive Path

Figure 3:

All A are B, Some C are B

• No direct path available
• Assumption: Reasoners change task structure to enforce a path
• NVC if path cannot be found
Figure 3:

- Premises with universal quantifiers (All, No) treated bidirectionally
- Yields same path structures as for Figure 1 and Figure 2 syllogisms
- Same mechanism for Figure 4 syllogisms
Determine Quantifier: Set Propagation

All A are B

Some B are C
Determine Quantifier: Set Propagation

All A are B

Some B are C
Determine Quantifier: Set Propagation

- All A are B
- Some B are C

A → A (B) → A (B,C)
Determine Quantifier: Set Propagation

- All A are B
- Some B are C

A → A (B) → A (B, C)
Ambiguity of “No” as first quantifier: Empty set vs “All A are no B”
- Empty set: No statement about elements of A
- “No A are B” interpreted as “All A are no B”

Set propagation fails
• Start from the end of the path
• Bidirectional interpretation if second premise quantifier is “All”
• Simplifies ambiguity and leads directly to the conclusion
Analysis

Comparison of models with most-frequent answer (MFA)

- MFA is the optimal response strategy for aggregate prediction models
- **Coverage**: Check if MFA is in set of possible model predictions
- **Accuracy**: Discount coverage score based on number of possible predictions
MFA Analysis

- TransSet achieves peak performance
- Cognitive models drop in performance when penalized for multiple responses
- Highlights unspecificity of model predictions
- Suggests severe shortcomings of the predictive forms of the models
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MFA Analysis

- TransSet achieves peak performance
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- Cognitive models drop in performance when penalized for multiple responses
  - Highlights unspecificity of model predictions
  - Suggests severe shortcomings of the predictive forms of the models
• Investigate how applicable reasoning strategies are to individual reasoners
• For individuals, evaluate the predictive accuracy on their responses (proportion of correct predictions)
• Heuristic models should be able to accurately predict a small number of participants and perform rather poorly on the rest
Complex models are unsuitable, unless they can fine-tune predictions.

Large variance of MFA predictions highlights the limit of aggregation-based strategies.

"Average reasoner" is an unsuitable representation for an individual.
Complex models are unsuitable, unless they can fine-tune predictions.

Large variance of MFA predictions
- Highlights the limit of aggregation-based strategies
- “Average reasoner” is an unsuitable representation for an individual
Conclusion

- TransSet is able to capture human reasoning data fairly well while adhering to known statistical effects and psychological phenomena:
  - Figural effect (Johnson-Laird, 1983)
  - Conversion (Chapman & Chapman, 1959)
  - Informativeness of quantifiers (Chater & Oaksford, 1999)
- Occam’s Razor: questions worth of complex fit-based models
  - Unnecessary for modeling syllogistic reasoning unless able to be fine-tuned to individuals
  - TransSet as a simple heuristic suffices for population-based aggregate predictions
References


Code on GitHub:
https://github.com/Shadownox/iccm-transset
Determine Quantifier

First Quantifier Positive?

Yes

In analogy to Atmosphere

Merge Quantifiers

Response

No

Second Quantifier All?

No

NVC

Yes

Possible Situations
A → B → C (Figure 1)
A ← B ← C (Figure 2)

Possible Situations
A → B ← C (Figure 3)
A ← B → C (Figure 4)

NVC

Fail if no universal premise available or if both premises feature the same quantifier

Impose path by reversing a universal premise in accordance to the preference order A > E

Yes

Impose path by reversing a universal premise in accordance to the preference order A > E

Determine Direction

Direct Path?

Yes

Direction Determined

No

Determine Direction

Model Flow