Implications of Guessing Types in Multinomial Processing Tree Models: Conditional Reasoning as an Example

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Abstract

Human responses in reasoning are sometimes based on guessing which is a cognitive process usually accounted for by adequate cognitive models. In the literature, different types of guessing have been reported but investigations of their impact on the overall model performance are rare.

This article focuses on three theories of conditional reasoning implemented as Multinomial Processing Trees (Oberauer, 2006). We analyze the impact of the different guessing types on the raw goodness of fit, on information criteria commonly found in the literature (AIC, BIC, FIA), discuss the partial influences of reasoning, guessing, and additional heuristic components, as well as assess the impact of guessing on parameter estimates. The results indicate that using different types of guessing can have a reliable impact on the model's performance and implications about the assumed cognitive processes.

Keywords: Cognitive Modeling; Conditional Reasoning; Multinomial Processing Trees; Guessing

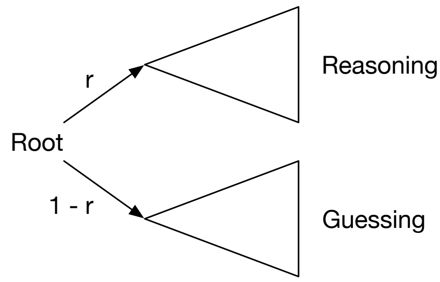


Figure 1: General structure of MPT models augmented with a guessing subtree. A parameter *r* represents the probability of entering the reasoning submodel, i.e., the process motivated by a cognitive theory.

Introduction

Computational modeling has recently gained popularity in a wide variety of research domains for its capability to evaluate and compare competing theoretical accounts in a well-defined manner. In cognitive science and psychology, theories are now routinely formalized as computational models such as *Multinomial Processing Trees* (MPTs; Riefer & Batchelder, 1988; Purdy & Batchelder, 2009). These models are of particular interest due to their inherent capability of modeling latent processes and have been used in a multitude of research domains such as memory storage and retrieval, perception, reasoning, or psychometrics (e.g., Batchelder & Riefer, 1999; Erdfelder et al., 2009). Even in the case of underspecified or verbally defined theories, they allow for a profound assessment of underlying assumptions.

Computational modeling is comprised of three phases: *model implementation*, *model evaluation*, and *model comparison*. While methods for model evaluation and comparison are standardized and available in various toolboxes (e.g., MPTinR; Singmann & Kellen, 2013), the development of models still offers considerable freedom to the modeler. In principle, there are (at least) two computational modeling approaches: First, cognitive computational models are developed to formalize (verbally specified) cognitive theories to evaluate their explanatory power (e.g., Oberauer, 2006). Second, the connections between identified cognitive processes and experimental variables can be systematically manipulated to develop a cognitively grounded theory (e.g., Klauer, Stahl, & Erdfelder, 2007).

Different goals have different requirements with respect to the modeling approaches being applied. Theory formalization requires the model to be as close to the original theoretical specification as possible. This largely prevents the modeler from incorporating additional assumptions targeted towards optimizing the performance of the resulting model. On the other hand, in general cognitive modeling, the modeler might decide on modifying the original theory in order to account for missing outcomes, or to make the resulting model comparable to a selection of alternative models with incompatible high-level process assumptions.

A model augmentation commonly found in the literature is a process usually coined "guessing". While technically representing a response generation process differing from regular inference, guessing is often incorporated into models as an alternative strategy to account for missing response categories in order to enable evaluation on general datasets or comparison with other candidate models (cf. Figure 1). Guessing can be based on different methods such as *uniform* approaches assigning equal probability to all outcomes, *bias* guessing assuming a bias for general acceptance of conclusions, or *independence* guessing specifying independent bias probabilities for individual inferences. Even though including a guessing component is a common asset in cognitive modeling, discussions of its influence and potential for negatively influencing obtained results are rarely found. Furthermore, previous research showed that sometimes guessing alone can achieve better performance than when paired with a reasoning part (Ragni & Tse, 2017). While this previous study evaluated the contribution of each inference pattern for conditional reasoning (with a bias guessing approach), the overall impact of different types of guessing has not been systematically analyzed yet.

Our goal is to add to the comprehensible and in-depth analysis conducted by Oberauer (2006). We extend it by analyzing the impact of different types of guessing on raw model performance with respect to goodness of fit, and a set of established information criteria (AIC, BIC, FIA), assessing the impact of guessing and a heuristically driven system, as well as evaluating the influence of guessing on parameter estimates. While we use the term *guessing* consistent with the literature in referring to the additional subtree, this does not imply that it can only reflect guessing processes and not alternative strategies such as heuristics. Our analysis compares the effects of three types of guessing: random guessing, a heuristically guided process and a theoretically motivated approach.

The article is structured as follows: In the next section we present our demonstrative domain of conditional reasoning and introduce three theories implemented as MPTs by (Oberauer, 2006). In Section 3, this set of models is augmented with different types of guessing and fit to data in order to analyze the impact of guessing on model performance and individual parameter estimates. A discussion about the impact of guessing types concludes the article.

State of the Art

In this section we briefly introduce conditional reasoning as our demonstrative domain and sketch the set of cognitive theories we base our analysis upon (Oberauer, 2006).

A conditional such as *if A then C* consists of an antecedent (here abbreviated by *A*) and a consequent (here abbreviated by *C*). If additional knowledge is given, such as *A*, then the consequent *C* can be inferred from the conditional (*Modus Ponens*, MP), for other additional information, such as not-*C*, the inference mechanism *Modus Tollens* (MT) can be applied to infer not-*A*. Both mechanisms are correct with respect to propositional logic. If instead the information *C* is given, the *Affirmation of Consequent* (AC) is sometimes applied by reasoners to infer *A*. If not-*A* is given the *Denial of Antecedent* (DA) can be applied to infer not-*C*. The last two mechanisms only make sense if the conditional is interpreted as a biconditional.

Human reasoners do deviate from the classical logical inferences (e.g., Klauer et al., 2007) and so cognitive theories have been developed that can better capture the human inference process than purely logical systems. Among them are the *Mental Model Theory* (MMT; Johnson-Laird, 1990; Johnson-Laird & Byrne, 2002) that assumes that human reasoners do reason with respect to a mental model of a conditional. A mental model is an interpretation of the given conditional with the initial mental model assuming that the antecedent and consequent are both true at the same time. While performing the reasoning task, other interpretations are possible and can be derived in a flesh-out process.

For his investigation of conditional reasoning, Oberauer (2006) implemented a set of models as *Multinomial Processing Trees* (MPTs; Riefer & Batchelder, 1988; Purdy & Batchelder, 2009). MPTs are a family of probabilistic models which can be used to model categorical data. Their core principle is based on the multinomial distribution and the assumption that observations are the product of a series of latent processes. By defining a tree structure on the set of assumed latent cognitive processes, MPTs can be used to test hypotheses related to human cognition. Model comparison is usually performed on the basis of so-called *Information Criteria* weighing the complexity of a model against the achieved fit to experimental data. The most prominent information criteria are *Akaike's Information Criterion* (AIC; Akaike, 1974), the *Bayesian Information Criterion* (BIC; Schwarz, 1978), and the Fisher Information Approximation (FIA; Rissanen, 1996).

Oberauer (2006) developed two models following the mental model theory as MPTs: one with directionality (i.e., with an additional assumption that mental models are processed from the antecedent to the consequent) and one without this additional assumption. These models define predictions for individual patterns on the basis of processes underlying the MMT account of reasoning, i.e., he presents MPTs that can predict an individual reasoners' choice of inference mechanisms from MP, AC, DA, MT.

Apart from MMT, Oberauer (2006) presents three further implementations of cognitive theories in form of MPTs: two variations of the *Suppositional Theory* (Evans & Over, 2004) which assumes that there are two reasoning processes, a fast and heuristic and a slower analytic one, as well as a *Dual-Process* (DP) approach (Verschueren, Schaeken, & d'Ydewalle, 2005) which combines inference on the basis of heuristic probability estimation with MMT.

Analysis of Guessing Types

Method

Guessing is understood as a way to produce answers "when reasoners are uncertain about the appropriate response but have to make a response nevertheless" (Klauer et al., 2007). In this sense, guessing represents an alternative strategy to reasoning for producing responses. It serves the purpose of explicitly representing human uncertainty as well as a general means of producing answers not accounted for by the theoretical account for reasoning.

Technically, guessing is a distribution over all possible responses. Depending on its degrees of freedom, it can represent true random guessing, or alternative strategies which may include biases or other strategies of varying complexity. In the following we investigate the effects of guessing based

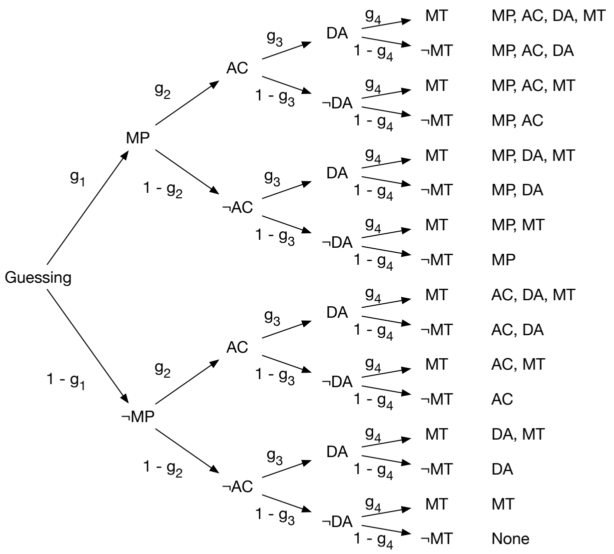


Figure 2: Guessing Subtree. Root node is reached with probability $(1-r)$. Parameters g_1, g_2, g_3, g_4 specify the probability distribution for reaching the 16 possible outcomes. ¬ indicates that the inference is not applied. This type of guessing corresponds to the independence model defined in Klauer et al. (2007).

on a set of three representative strategies taken from recent literature: *Bias Guessing*, *Independence Guessing*, and *Uniform Guessing*.

Bias Guessing. In his original paper, Oberauer (2006) augmented the theoretical accounts of conditional reasoning with guessing subtrees consisting of a single free parameter g representing a bias of accepting any of the four inference types (MP, AC, DA, MT). In this sense, bias guessing represents a basic strategy defined by a single probability parameter representing a reasoner's bias towards applying an inference without relying on reasoning processes. The MPT representing this type of guessing can be obtained from Figure 2 by setting the parameters equal: $g = g_1 = g_2 = g_3 = g_4$. By multiplying the parameters of a certain branch, the corresponding outcome probability is computed. For instance, the pattern (MP, DA, MT) has the probability $P((\text{MP, DA, MT})) = g \times (1-g) \times g \times g$.

Independence Guessing. A different way of handling guessing has been used as part of the *Inference-Guessing* model for conditional reasoning (Klauer et al., 2007). Originally devised for the Wason Selection task (Wason, 1968), this model employs a guessing strategy which assigns individual probability parameters (g_1, g_2, g_3, g_4) to each inference. In a sense, it extends on bias guessing by introducing independent biases for each inference increasing its capability to adapt to observed data. The MPT submodel for independence guessing is depicted in Figure 2. Pattern (MP, DA, MT) is assigned the probability $P((\text{MP, DA, MT})) = g_1 \times (1-g_2) \times g_3 \times g_4$.

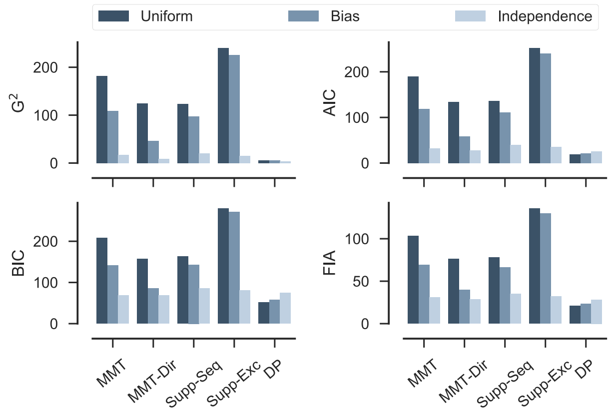


Figure 3: Visualization of the relations between the different metrics.

Uniform Guessing. Uninformed guessing can be represented by a uniform probability distribution over the set of outcomes. In the case of conditional reasoning with 16 different inference patterns producing a binary guessing tree, this corresponds to a fixed g parameter of $g = g_1 = g_2 = g_3 = g_4 = 1/2$. Each pattern is therefore assigned the probability $P(x) = g^4 = 1/16$.

Note, that uniform guessing does not relate processing paths to their corresponding outcomes. Instead it just uniformly assigns probability mass to the set of potential conclusions. This raises the question which properties of guessing differentiate between guessing and alternative reasoning strategies. One possibility to define guessing in a clear distinction from alternative reasoning strategies could be by focusing on context-dependency. If guessing processes are affected directly by the context of the task being modeled, e.g., by the premise information for conditional reasoning, it might be more accurate to refer to them as (heuristic) strategies. Arguably, this notion then ties into the framework of dual-process models (e.g., Evans, 1984).

Results

We analyze the set of models created by combining the conditional reasoning MPTs with the three guessing strategies on the dataset for "basic conditionals" reported by Oberauer (2006). This data was originally obtained by conducting an online study where 343 participants assessed the validity of conditional inferences. Model fits were computed via `MPTinR` (Singmann & Kellen, 2013), a state-of-the-art framework for evaluating MPT models using the `R` environment for statistical computation (R Core Team, 2014). Goodness of fit results as well as the information criteria AIC, BIC, and FIA are reported as produced by the MPTinR analysis.

Table 1: Fitting results of the model-guessing combinations and guessing alone (None).

Cognitive Theory	Guessing	Log Likelihood	G^2	AIC	BIC	FIA	Parameters
None	Uniform	-1902.00	1120.90	1120.90	1120.90	-	0
	Bias	-1856.19	1029.28	1031.28	1035.81	518.82	1
	Independence	-1495.85	308.60	316.60	334.72	168.26	4
MMT	Uniform	-1432.72	182.34	190.34	208.46	103.28	4
	Bias	-1395.82	108.55	118.55	141.20	69.12	5
	Independence	-1349.92	16.75	32.75	69.00	30.97	8
MMT-Dir	Uniform	-1403.68	124.27	134.27	156.92	76.03	5
	Bias	-1364.76	46.43	58.43	85.62	39.79	6
	Independence	-1346.23	9.37	27.37	68.15	28.33	9
Supp.-Sequential	Uniform	-1403.45	123.82	135.82	163.00	77.76	6
	Bias	-1390.26	97.44	111.44	143.15	66.22	7
	Independence	-1351.76	20.42	40.42	85.73	35.04	10
Supp.-Exclusive	Uniform	-1461.70	240.30	252.30	279.49	135.54	6
	Bias	-1454.50	225.90	239.90	271.61	129.96	7
	Independence	-1349.33	15.57	35.57	80.88	32.35	10
Dual Process (DP)	Uniform	-1344.32	5.54	19.54	51.26	20.99	7
	Bias	-1344.32	5.54	21.54	57.79	23.05	8
	Independence	-1343.21	3.33	25.33	75.17	28.34	11

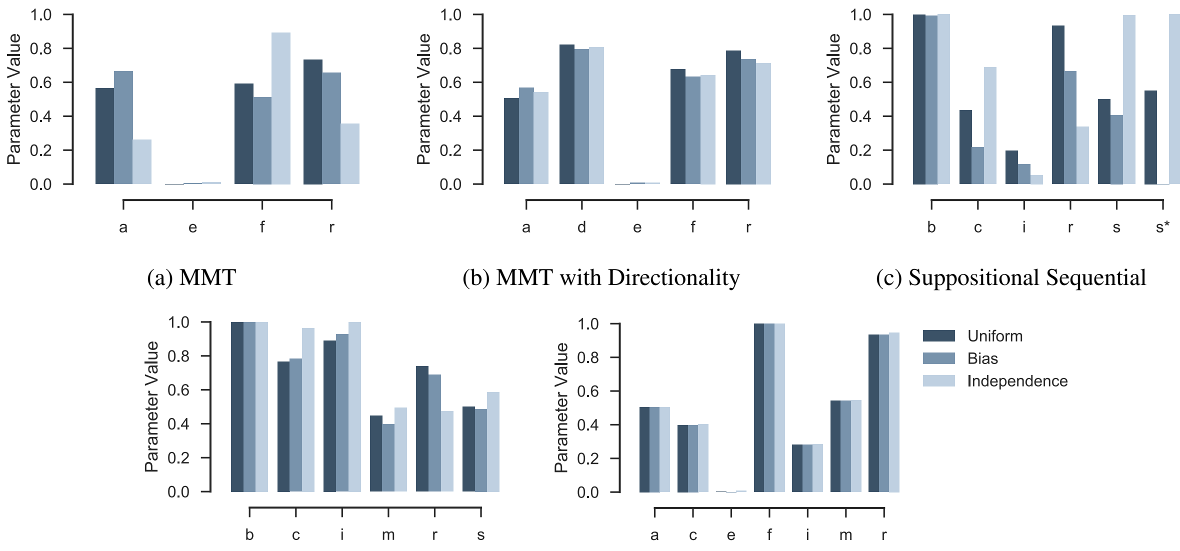
Table 1 and Figure 3 depict the results obtained from fitting the set of models to the data. Apart from the fits of the combined model, Table 1 also contains the results produced by fitting the guessing subtrees alone. Due to the fact that the theories themselves do not account for the complete set of possible outcomes for the conditional reasoning task, the performance metrics without guessing parts could not be determined.

The resulting values illustrate that the choice of guessing has a substantial impact on the overall model performance. Depending on the type of guessing, a wide range of values is obtained. Independence guessing leads to the best performing models, followed by bias guessing, and lastly uniform guessing as the worst option when considering optimality of the fit alone. These results are to be expected as they follow the number of degrees of freedom the guessing strategies add to the model. This is also reflected by the distance between the results of the different types of guessing. Bias guessing, which has only one free parameter is much closer to uniform guessing than independence guessing which features three free parameters. The only exception to this behavior is constituted by the Dual-Process (DP) model, which produces the overall best results and appears to be affected less severely by guessing. However, the comparatively superior goodness of fit results and insignificant variation of the three guessing types suggest that an upper bound of performance is reached. In consequence, the penalty terms of AIC, BIC, and FIA have a bigger impact on the information criteria values.

The guessing models by themselves result in the worst accounts for the data. This is not surprising since guessing does not contain theoretically motivated assumptions about cognition. Instead, these models represent uninformed strategies for producing responses to the task being modeled. Still, when being used as additions to formalized cognitive theories, they are capable of positively influencing the resulting model's performance. By accounting for responses not matching the underlying theory's implications, the predictive power of the theory is enhanced.

Figure 4 illuminates the effects of guessing from the perspective of individual parameter estimates. It shows that the different types of guessing influence the parameter estimates obtained from the fitting procedure. For instance, when considering MMT, the a parameter varies between a value of 0.56 for uniform guessing, 0.66 for bias guessing, and 0.26 for independence guessing. The magnitude of variance observed shows that guessing needs to be applied cautiously when aiming at interpreting the cognitive processes represented by the model parameters. However, the plots also illustrate that the impact guessing has on parameter estimates is dependent on the model itself. MMT with directionality and the dual process model appear to be much less influenced than MMT or the sequential suppositional model, for instance.

Of particular importance is the parameter r which represents the probability of entering the reasoning part of the model instead of relying on guessing for producing a response. Considering the values of r, depicted isolatedly from the other parameters in Figure 5, a considerable influence of guessing can be observed for most of the models. The reason-



(a) MMT (b) MMT with Directionality (c) Suppositional Sequential

(d) Suppositional Exclusive (e) Dual Process

Figure 4: Parameter estimates resulting from fitting the set of models with different guessing trees to the data.

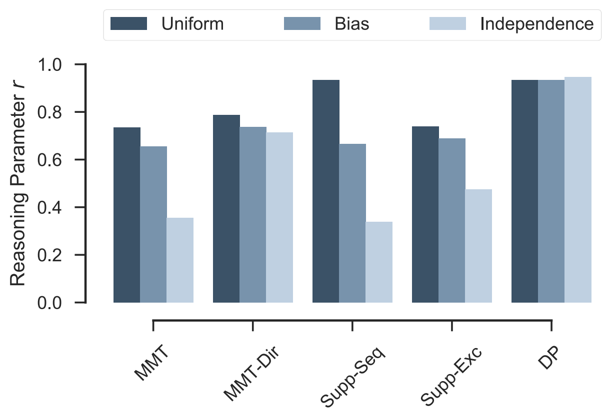


Figure 5: Reasoning parameter r resulting from fitting the theory-guessing model combinations.

ing parameter r differs between 0.73 for uniform and 0.36 for independence guessing. Put differently, by simply switching the type of guessing, the underlying theory is 37% less likely to account for the data. This adds to the observation indicating that the impact of guessing follows the degrees of freedom of the respective strategies. If guessing features larger numbers of free parameters, it is able to account for larger proportions of the data, reducing the importance of the actual reasoning component.

General Discussion

Implementing cognitive theories has become a core aspect of cognitive science. Apart from the raw goodness of fit metrics, interpretability and theoretical merit are essential factors to judge models by. However, when implementing models, the need to add assumptions unwarranted by the underlying theoretical foundation frequently arises. This obfuscates the true power of the theory and may lead to a distortion of resulting qualitative assessments.

Our results show that even seemingly unintrusive additions such as the addition of guessing processes not accounted for by the underlying theory may have unexpectedly high impact on the overall model performance. A shift in the performance of the models following the degrees of freedom available in the guessing trees can be observed. Uniform guessing is not able to be adapted for an optimal fit to the data resulting in the most explanatory weight being assigned to the underlying theory which is reflected by the worst performance values but a relatively high probability of entering the reasoning part of the model. In contrast, bias and independence guessing represent theoretically motivated strategies as alternatives to reasoning. By offering one or three parameters, respectively, for fitting the model, higher levels of performance are achieved at the cost of larger proportions of the data being accounted for by guessing.

When evaluating theoretical accounts on the basis of model implementations, special care needs to be taken to disentangle the original theory's performance from the influence of the additional assumptions. Our results illustrate that there is a fine line between guessing and what must be considered alternative strategies to reasoning. Even when introducing additional processing paths based on a single additional parameter, a hybrid model is formed which produces results that cannot be attributed solely to the underlying theoretical concepts. By disregarding the need for theoretical justification due to treating those alternative strategies simply as "guessing", their potential intricacies, dependencies to the data, and thus influence on the theoretical foundation are obscured.

The results of this work can be generalized to other modeling tasks. Regardless of the framework in use, the addition of alternative processing paths to producing conclusions have an impact on the overall model performance. Without ensuring that the modifications only result in controlled local effects, the soundness of the underlying theoretical assumptions cannot be expected to remain intact. As a conclusion, the role of the model as a representative instance for a theoretical account becomes debatable.

Our findings raise the question as to whether parameterized guessing components can be understood as *guessing* in the first place. Instead, it might be more appropriate to distinguish pre-determined probability distributions with no degrees of freedom as guessing and parameterized versions as (sometimes implicit) realizations of dual-process models with a representation of heuristics. Consequently, guessing might be better defined in terms of context-independent processes that do not depend on the presented information. The present analysis demonstrates the impact of the different types of guessing on reasoning. It highlights the need for a comprehensive theory of guessing.

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